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Kelas : Matematika 1 (4)

1. Determinan ukuran $3 \times 3 \rightarrow$ No. 2a hal 57

$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 7 & 8 \\ 2 & 9 & 6 \end{pmatrix} \quad B_2 - 2B_1 \quad \begin{pmatrix} 1 & 3 & 5 \\ 0 & 1 & -2 \\ 0 & -3 & -4 \end{pmatrix}$$

$$\rightarrow \begin{vmatrix} 1 & 3 & 5 \\ 0 & 1 & -2 \\ 0 & -3 & -4 \end{vmatrix} = 1 \begin{vmatrix} 1 & -2 \\ 3 & -4 \end{vmatrix} - 0 \begin{vmatrix} 3 & 5 \\ 3 & -4 \end{vmatrix} + 0 \begin{vmatrix} 3 & 5 \\ 1 & -2 \end{vmatrix}$$

$$= 1(-4 - (-6))$$

$$= 1(2)$$

$$= 2 //$$

Jadi, determinan matriks tersebut adalah 2

2. Determinan ukuran $4 \times 4 \rightarrow$ No. 2D hal 58

$$\begin{pmatrix} 1 & 4 & 3 & 1 \\ 1 & 5 & 6 & 3 \\ 2 & 4 & 8 & 8 \\ 1 & 4 & 6 & 2 \end{pmatrix} \quad \begin{matrix} B_2 - B_1 \\ B_3 - 2B_1 \\ B_4 - B_1 \end{matrix} \quad \begin{pmatrix} 1 & 4 & 3 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & -4 & 2 & 6 \\ 0 & 0 & 3 & 1 \end{pmatrix}$$

$$\rightarrow \begin{vmatrix} 1 & 4 & 3 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & -4 & 2 & 6 \\ 0 & 0 & 3 & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & 3 & 2 \\ -4 & 2 & 6 \\ 0 & 3 & 1 \end{vmatrix} - 0 + 0 - 0$$

$$= 1 \begin{vmatrix} 2 & 6 \\ 3 & 1 \end{vmatrix} - (-4) \begin{vmatrix} 3 & 2 \\ 3 & 1 \end{vmatrix} + 0$$

$$= -16 + 4(-3)$$

$$= -16 - 12$$

$$= -28 //$$

Jadi, determinan matriksnya adalah -28

3. SPL $3 \times 3 \rightarrow$ Metode Cramer $\Rightarrow$

$$\text{Persamaan : } 2x + 5y - 3z = 3$$

$$6x + 8y - 5z = 7$$

$$-3x + 3y + 4z = 15$$

$$\rightarrow \begin{pmatrix} 2 & 5 & -3 \\ 6 & 8 & -5 \\ -3 & 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ 15 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 5 & -3 \\ 6 & 8 & -5 \\ -3 & 3 & 4 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 3 & 5 & -3 \\ 7 & 8 & -5 \\ 15 & 3 & 4 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 2 & 3 & -3 \\ 6 & 7 & -5 \\ -3 & 15 & 4 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 2 & 5 & 3 \\ 6 & 8 & 7 \\ -3 & 3 & 15 \end{pmatrix}$$

$$x_1 = \frac{\det(A_1)}{\det(A)}$$

$$\begin{aligned} \rightarrow \det(A) &= \begin{vmatrix} 2 & 5 & -3 \\ 6 & 8 & -5 \\ -3 & 3 & 4 \end{vmatrix} \xrightarrow{B_2 - 3B_1} \begin{vmatrix} 2 & 5 & -3 \\ 0 & -7 & 4 \\ -3 & 3 & 4 \end{vmatrix} = 2 \begin{vmatrix} -7 & 4 \\ 3 & 4 \end{vmatrix} - 0 + (-3) \begin{vmatrix} 5 & -3 \\ -7 & 4 \end{vmatrix} \\ &= 2(-28 - 12) + (-3)(20 - 21) \\ &= 2(-40) + 3 \\ &= -77 \end{aligned}$$

$$\begin{aligned} \rightarrow \det(A_1) &= \begin{vmatrix} 3 & 5 & -3 \\ 7 & 8 & -5 \\ 15 & 3 & 4 \end{vmatrix} = 3 \begin{vmatrix} 8 & -5 \\ 3 & 4 \end{vmatrix} - 7 \begin{vmatrix} 5 & -3 \\ 3 & 4 \end{vmatrix} + 15 \begin{vmatrix} 5 & -3 \\ 8 & -5 \end{vmatrix} \\ &= 3(32 + 15) - 7(20 + 9) + 15(-25 + 24) \\ &= 3(47) - 7(29) + 15(-1) \\ &= 141 - 203 - 15 \\ &= -77 \end{aligned}$$

$$\begin{aligned} \rightarrow \det(A_2) &= \begin{vmatrix} 2 & 3 & -3 \\ 6 & 7 & -5 \\ -3 & 15 & 4 \end{vmatrix} \xrightarrow{B_2 - 3B_1} \begin{vmatrix} 2 & 3 & -3 \\ 0 & -2 & 4 \\ -3 & 15 & 4 \end{vmatrix} = 2 \begin{vmatrix} -2 & 4 \\ 15 & 4 \end{vmatrix} + (-3) \begin{vmatrix} 3 & -3 \\ -2 & 4 \end{vmatrix} \\ &= 2(-8 - 60) + (-3)(12 - 6) \\ &= 2(-68) + (-3)6 \\ &= -136 - 18 \\ &= -154 \end{aligned}$$

$$\begin{aligned} \rightarrow \det(A_3) &= \begin{vmatrix} 2 & 5 & 3 \\ 6 & 8 & 7 \\ -3 & 3 & 15 \end{vmatrix} \xrightarrow{B_2 - 3B_1} \begin{vmatrix} 2 & 5 & 3 \\ 0 & -7 & -2 \\ -3 & 3 & 15 \end{vmatrix} = 2 \begin{vmatrix} -7 & -2 \\ 3 & 15 \end{vmatrix} + (-3) \begin{vmatrix} 5 & 3 \\ -7 & -2 \end{vmatrix} \\ &= 2(-105 + 6) + (-3)(-10 + 21) \\ &= 2(-99) + (-3)(11) \\ &= -198 - 33 \\ &= -231 \end{aligned}$$

$$x = \frac{\det(A_1)}{\det(A)} = \frac{-77}{-77} = 1 ; y = \frac{\det(A_2)}{\det(A)} = \frac{-154}{-77} = 2 ; z = \frac{\det(A_3)}{\det(A)} = \frac{-231}{-77} = 3$$

Jadi, nilai x adalah 1, nilai y adalah 2, dan nilai z adalah 3

4. Nilai Eigen $2 \times 2 \rightarrow$ No 29 halaman 67

$$\begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$$

$$Ax = \lambda x$$

$$Ax - \lambda x = 0$$

$$(A - \lambda I)x = 0$$

$$\rightarrow \det(A - \lambda I) = 0$$

$$\begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \\ = \begin{pmatrix} 2 - \lambda & 1 \\ 2 & 3 - \lambda \end{pmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 2 - \lambda & 1 \\ 2 & 3 - \lambda \end{vmatrix} = 0$$

$$= (2 - \lambda)(3 - \lambda) - 2 \cdot 1 = 0$$

$$6 - 5\lambda + \lambda^2 - 2 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$(\lambda - 4)(\lambda - 1) = 0$$

$$\lambda = 4 \vee \lambda = 1$$

$$\text{Saat } \lambda = 4 \rightarrow \begin{pmatrix} 2 - \lambda & 1 \\ 2 & 3 - \lambda \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$-2x_1 + x_2 = 0$$

$$2x_1 = x_2, \text{ misal } x_1 = 5$$

$$\rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} = 5 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\text{Saat } \lambda = 1 \rightarrow \begin{pmatrix} 2 - \lambda & 1 \\ 2 & 3 - \lambda \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= x_1 + x_2 = 0$$

$$x_1 = -x_2, \text{ misal } x_1 = 5$$

$$\rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \end{pmatrix} = 5 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Jadi, nilai eigennya adalah $\lambda = 4$ atau $\lambda = 1$

Saat $\lambda = 4$, vektor eigennya = $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ dan saat $\lambda = 1$, vektor eigennya = $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

5. Nilai eigen matriks $3 \times 3 \rightarrow$ no. 2d hal 67

$$\begin{pmatrix} 1 & -1 & 5 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix}$$

$$Ax = \lambda x$$

$$(A - \lambda I)x = 0$$

$$\det(A - \lambda I) = 0$$

$$\begin{pmatrix} 1 & -1 & 5 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} = \begin{pmatrix} 1-\lambda & -1 & 5 \\ 0 & 2-\lambda & 5 \\ 0 & 0 & 3-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = (1-\lambda) \begin{vmatrix} 2-\lambda & 5 \\ 0 & 3-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)(3-\lambda) = 0$$

$$\lambda = 1 \vee \lambda = 2 \vee \lambda = 3$$

$$\text{Saat } \lambda = 1 \rightarrow \begin{pmatrix} 0 & -1 & 5 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-x_2 + 5x_3 = 0$$

$$x_2 = 5x_3 \rightarrow x_2 = 5t$$

$$x_2 + 5x_3 = 0$$

$$2x_3 = 0$$

$$x_3 = t = x_1$$

$$x = \begin{bmatrix} t \\ 5t \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$$

$$\text{Saat } \lambda = 2 \rightarrow \begin{pmatrix} -1 & -1 & 5 \\ 0 & 0 & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-x_1 - x_2 + 5x_3 = 0 \quad x_3 = t \quad x_1 + x_2 = 5t$$

$$5x_3 = 0$$

$$x_2 = 5 \rightarrow x_1 = 5t - 5$$

$$x_3 = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} \rightarrow x_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad x_2 = \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{Saat } \lambda = 3 \rightarrow \begin{pmatrix} -2 & -1 & 5 \\ 0 & -1 & 5 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-2x_1 - x_2 + 5x_3 = 0$$

$$x_1 = 5$$

$$-x_2 + 5x_3 = 0$$

$$x_2 = 5x_3$$

$$x_3 = t \rightarrow x_2 = 5t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Jawab: Nilai eigen dari matriks 3×3 tersebut adalah $\lambda = 1 \vee \lambda = 2 \vee \lambda = 3$